Introduction to Game Theory	Date: *April, 2020*
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Modelling the stock market using game theory

1 Introduction

A stock exchange is an extremely complex, self organising financial system. In a stock exchange, a company can enlist to sell it's shares to people (investors) in exchange for money. The company can then use that money to accelerate it's business interests, and if it profits (it's valuation goes up), the price of it's stocks increases. The investor (player/agent, as we'll later see) can sell his stock holdings at any time, for the current value of the stock. If the selling price is higher than the price at which he bought the stock (i.e., the company has performed well and people have confidence in it's stock) then the agent will profit. But the stock market has a reputation to be extremely volatile and fast-paced. The complexity is introduced because of the vast amount of possibilities that arise as the number of players (investors) and stocks increases. The governing policies, company rules and competition further increases the complexity. The amount of information available to analyse to determine an efficient strategy is extremely high, and hence the task is extremely difficult. But to understand the underlying game in any trading situation, we'll model a simple situation and then try to extend it further.

1.1 The stock market environment

The stock market is a highly dynamic & complex system with various factors constantly changing stock prices. The main one that affects the stock price is the supply and demand in the market. Other factors include the money earned by a company from its product/service. Different investors use different approaches to investing: fixed patterns, algorithms, AI, intuition, etc. Modelling it as a game can help give an agent on how to go about investing and is in fact used by many firms nowadays.

1.2 Game Theory & Stock Market

The stock market decisions everyday investors or speculators make in terms of different investment strategies and different players (investors). Since each agent has so many options it is unlikely that one option or strategy consistently leads to the highest payoff (a dominant strategy). However an agent can still try to figure out the best response strategy for a given situation by modelling the current market as a game.

2 Model

To define a strategic form game $\Gamma = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N}\rangle$ in this case, consider the following definitions:

- $N = \{1, 2, \cdots n\}$ is the finite set of players (major investors)
- $J = \{1, 2, \cdots j\}$ is the finite set of companies offering stocks
- $Z(t) = \{z^1(t), z^2(t), \cdots z^j(t)\}$ is the set of price functions, where $z^i(t)$ is the price of the i^{th} stock at a time t.

- $S_i(t) = \{s_i^1(t), s_i^2(t), \cdots s_i^j(t)\}$ is the set of strategies, where $s_i^j(t)$ is the number of shares of the company j sold by the player i at time t at a price $z^i(t)$.
- $u_i(T) = \sum_{t=1}^{T} \sum_{k=1}^{j} s_i^k(t) * z^k(t)$ is the utility function, such that $u_i(T)$ represents the total profit of the player i by the time T.
- If the utility is defined over a continuous variable of time, then we can use the following definition (the functions have meanings as defined above):

$$u_i(T) = \sum_{k=1}^{j} \int_{t=0}^{T} s_i^k(t) * z^k(t)$$

3 Strategies

The complexity of the market makes it an extremely hard game to analyse. But in a simplified model of a stock market, an individual agent can be thought of as playing against a majority of the other agents, making it a two player game. As will be clear from the examples below, there are no PSNEs. This model is useful because the movement of the stock prices actually depends on what the majority of the agents thinks of the company or the stock. But as in the example below, there is a mixed strategy equilibrium.
In reality, game theory by definition is the strategic interaction between between rational and self interested agents. Here, the rules of the game (governing policies) are such that a lot of strategies are illegal. For example, buying/selling of stocks in large amounts using information not in the public domain (common knowledge) is considered insider trading, and is illegal. In reality, any action by which an individual can *significantly* change the state of the market is illegal. We would argue that the market is actually designed in this manner to meet the requirements for stability and growth in the economy.

4 Example

4.1 Go long or short against a majority

Investment in the market can be simply modelled as a game against a majority. Let's say we want to decide if we want to sell (go 'short' in the common tongue) or retain (go 'long') some amount of shares of a company $\mathcal{A}$. If you decide to go long and a majority of other investors also decide to go long, we will profit. If we agree with overall market sentiment, we will profit in the short-term. Otherwise, we will lose in the short-term. The payoff matrix is as shown in the table below, where we are the row player, and the majority of other investors are the column player. But it should be noted that the simplification here yields that the payoff of the column player is distributed among all the other players, while our payoff is for us to keep.

Table 1: Payoff Matrix

	Long	Short
Long	(10, 10)	(-20, 40)
Short	(-20, 40)	(10, 10)

As is clear from the payoff matrix above, even in this simple situation, there is no pure strategy Nash equilibrium (PSNE) or a dominant strategy (DS). Profit/loss depends on how well we can predict the investment decisions of the majority of other investors. In a sense, we are not playing against other investors, but against a majority. But, there is a mixed strategy equilibrium in this situation. It can be arrived at in the following manner:

Let q denote the probability that majority of investors will go long. Then $1 - q$ is the probability they will go short. Let p is the probability that we go long. Then $1 - p$ is the probability that we go short.

$$u_1 = p * q * (10) + p * (1 - q) * (-20) * (1 - p) * q * (-20) + (1 - p) * (1 - q) * (10)$$

$$\Rightarrow u_1 = 10 + 20 * p * q - 30 * p - 30 * q \tag{1}$$

Similarly,

$$u_2 = 10 - 20 * p * q + 30 * p + 30 * q \tag{2}$$

Hence, we see that the mixed strategy Nash equilibrium is that we go long half the time and the majority of investors go long half the time. Therefore, allowing randomisation gives us better strategies.

5 Challenges faced in modelling

The most obvious factors to take into account would be solid numbers: stock price, company valuation, etc. But in the real world there would be a number of intangible factors that could affect stock price unexpectedly. For example, the reputation of a company as well as the key members (the board, executives, etc.) play a major role, and that acts as common knowledge in the game. It would affect whether an investor(agent) would invest in the company or not or the amount of money invested. This in turn changes the stock price. Situations like these are extremely hard to quantitatively take into account. There are many other possibilities such as launch of a new product, economic crisis, the market taking a major turn that could unexpectedly change the stock prices and present challenges in modelling.