Quantum Computational Advantage Using Photons

At least what my CS brain could comprehend 🐵

Aditya Morolia

May 2, 2026

Roadmap

1. Why boson sampling is a meaningful complexity-theoretic target.
2. Why boson sampling is hard to scale.
3. Gaussian boson sampling
4. The Jiuzhang machine

Context

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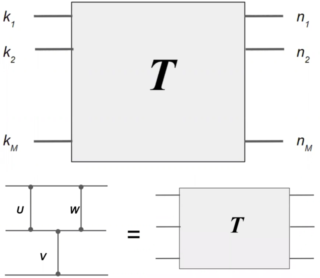
- Boson sampling: non-universal task, designed to violate this.
- Idea: sample from some probability distribution that is easy to generate with photons but #P-hard to simulate classically.

Boson Sampling



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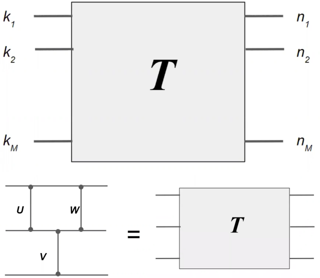

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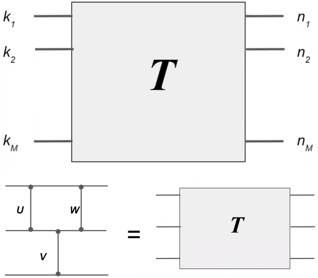

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- Output: $(n_1, \ldots, n_m)$, the occupation numbers of the output modes.
- Output probabilities are governed by permanents of submatrices of T:

$$\Pr(n \mid k, T) = \frac{|\mathrm{Perm}(T_{n,k})|^2}{\prod_i^m n_i! k_i!}.$$

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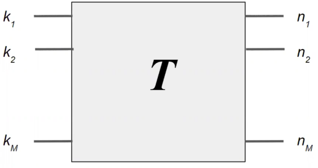

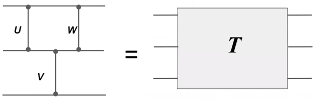

$$\begin{pmatrix} T_{1,1} & T_{1,2} & T_{1,3} & T_{1,4} \\ T_{2,1} & T_{2,2} & T_{2,3} & T_{2,4} \\ T_{3,1} & T_{3,2} & T_{3,3} & T_{3,4} \\ T_{4,1} & T_{4,2} & T_{4,3} & T_{4,4} \end{pmatrix} \quad \mathbf{T_{n,k}} = \begin{pmatrix} T_{1,1} & T_{1,2} & T_{1,3} \\ T_{3,1} & T_{3,2} & T_{3,3} \\ T_{3,1} & T_{3,2} & T_{3,3} \end{pmatrix}$$

When
$n = (1, 0, 2, 0), k = (1, 1, 1, 0)$.

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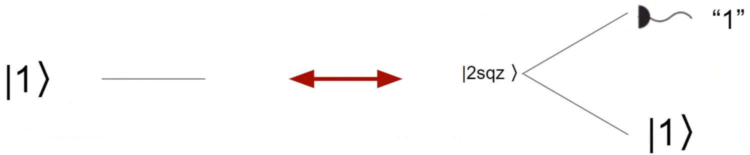
- Standard boson sampling wants many near-deterministic, indistinguishable single photons.

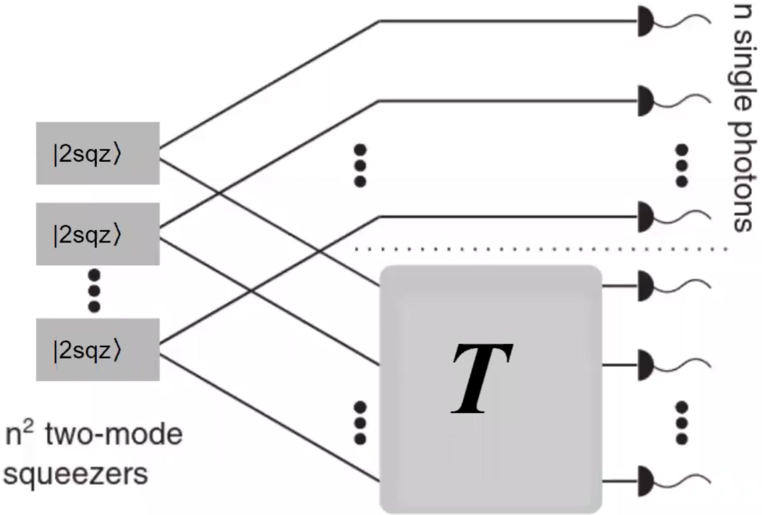
Beyond Standard Boson Sampling



$$\begin{aligned} |2\text{sqz}\rangle &= \exp\left(r a^\dagger b^\dagger - \text{H.c.}\right)|00\rangle \\ &= \frac{1}{\cosh r}\exp\left(\tanh r\, a^\dagger b^\dagger\right)|00\rangle \\ &= \sum_{n=0}^{\infty}\frac{\tanh^n r}{\cosh r}|nn\rangle \end{aligned}$$

Outline of SPDC source.

Scattershot Boson Sampling



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- With threshold detectors, one-pair and multi-pair events look the same
- So you must work in a very weak-pumping regime where clean single-photon events are rare.
- Why early boson-sampling experiments stayed in low-photon regimes even before worrying about interferometers and detectors.
- You are trying to impose high indistinguishability, low loss, deep interferometers, and stable long-time operation all at once.

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$$\Pr(\mathbf{n} \mid T, \{r_i\}) = C \frac{|\mathrm{haf}(B_{\mathbf{n},\mathbf{n}})|^2}{\prod_{i=1}^m n_i!}$$

$$\mathrm{haf}(A) = \frac{1}{n! 2^n} \sum_{\sigma \in S_{2n}} \prod_{j=1}^{n} A_{\sigma(2j-1),\sigma(2j)}$$

$$B = T \left[\bigoplus_{i=1}^{M} \tanh r_i \right] T^\top$$

Note: Using threshold detectors, the the output distribution is related to the Torontonian.

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Squeezed vacuum $\sum_{k=0}^{\infty} g(k) e^{ik\phi} |2k\rangle$

$$P_n = \left| \sum \text{all possible photon numbers} \left(\sum \text{all possible paths} \right) \right|$$

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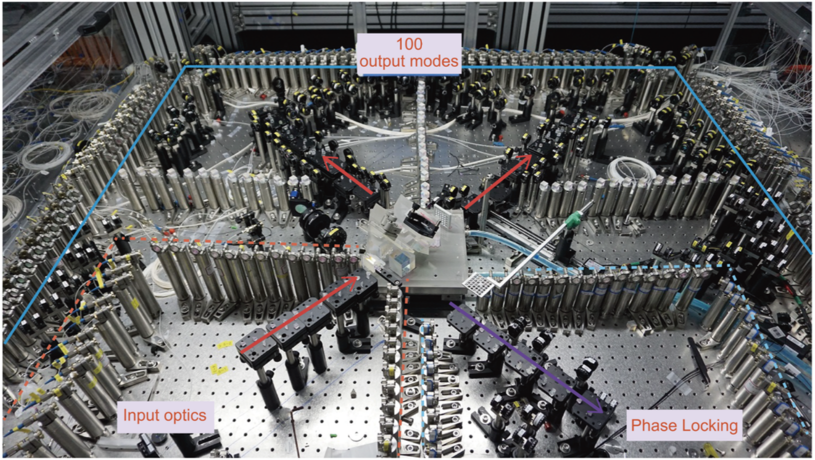
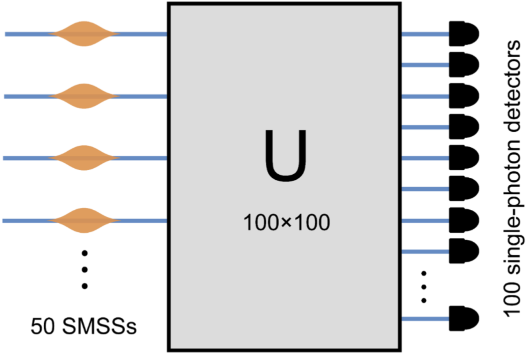
5. **Validation and benchmarking.** Generation not enough. Samples are sparse points in an enormous output space. Must rule out classical impostors and compare against strong classical simulators.

Jiuzhang: A Phase-Locked Gaussian Boson Sampler





Supplementary Figure S1 | An overview of our GBS implementation. The GBS device is realized by sending 50 single-mode squeezed states (SMSSs) into a 100-mode interferometer, and then sampling from the output distribution using 100 single-photon detectors.

Light Source

- A PPKTP source produces a two-mode squeezed state (TMSS) in modes a and b:

$$S_{ab}(r) = \exp\left[r\left(a^\dagger b^\dagger - ab\right)\right], \qquad |\text{TMSS}\rangle = S_{ab}(r)\,|00\rangle\,.$$

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- So one TMSS is unitarily equivalent to two independent single-mode squeezed states:

$$S_{ab}(r) = S_c(r) \otimes S_d(-r).$$

  One mode is squeezed by r, the other by $-r$.

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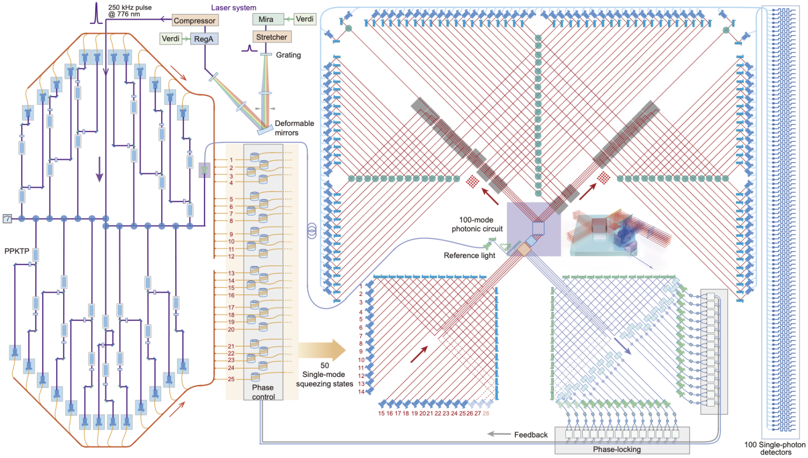
- The interferometer then uses hybrid encoding:

$$\text{50 spatial modes} \times \text{2 polarizations} = \text{100 modes.}$$



Supplementary Figure S21 | A schematic diagram of GBS experimental setup. The system can be devided into four parts: squeezed light sources, phase locking system, interferometer and single-photon detectors. First, a transform-limited pulse laser with an overall power of 1.4 W are equally divided into 13 paths to pump 25 independent PPKTP crystals.The generated collinear two-mode squeezed light in each path is collected into a single-mode fiber. To actively stabilize the phase of each squeezed light source, a 5-m fiber is winded around a piezo-electric (PZT) cylinder to compensate the phase change so that we can stabilize the phase in a fixed point with PID feedback control. Then, 25 phase-locked TMSSs lights are injected into a 100-mode interferometer to talk with each other. Finally, the output state are detected by 100 superconducting nanowire single-photon detectors.

Phase Locking

- Restore the phase dependence of the input squeezed states:

$$
\begin{aligned}
\xi_i &= r_i e^{i\phi_i}, \\
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$$\phi_i \mapsto \phi_i + \phi_0 \ \forall i \Longrightarrow B \mapsto e^{i\phi_0} B,$$

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- Relative phase evolution is the real problem:

$$\left| \sum_{\mu} a_\mu e^{i\theta_\mu(t)} \right|^2 = \sum_{\mu} |a_\mu|^2 + \sum_{\mu \neq \nu} a_\mu a_\nu^* e^{i(\theta_\mu(t) - \theta_\nu(t))}.$$

If the $\phi_i(t)$ is not constant, the second sum averages toward zero.

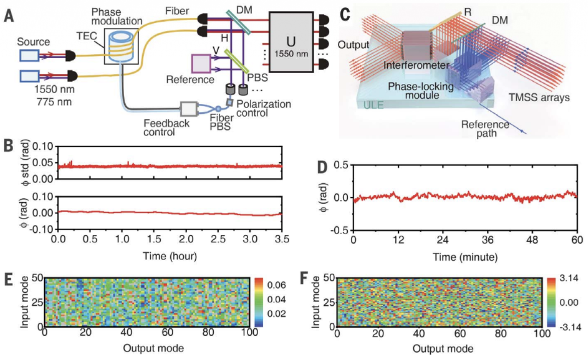

Fig. 2. Phase locking from the photon sources to the interferometer. (**A**) Schematic diagram of the active phase-locking system. A pump laser beam is used as a reference for all the squeezed states. After propagating through a ~2-m free space and 20-m optical fiber, a ~10-µW pump laser that shares the same propagation path as the downconverted photons is separated by a dichromatic mirror; The pump laser pulses are then combined on a beamsplitter with the reference laser pulse. A balanced detection scheme, which is insensitive to laser power fluctuation, is used to read out the phase information. To overcome the path length fluctuation, we wind optical fiber (length 5 m) around a piezoelectric cylinder with sensitivity of 1.5 rad/V, resonance frequency of 18.3 KHz, and dynamical range of 300 rad. (**B**) Phase stability tests. The top and bottom panels respectively show a typical monitoring of phase fluctuation of active and passive phase locking over 3.5 hours. The measured standard deviation of the phase is as small as 0.02 rad (λ/150) for active phase locking and 0.017 rad (λ/180) for passive phase locking. (**C**) We apply passive phase stabilization to the interferometer by adhering the devices onto an ultralow-expansion glass plate that is temperature-stabilized within 0.02°C. The blue light paths are for the interference of the 25 pumping lasers with the reference laser. The red light paths are the input and output of the photonic network. R denotes a reflective mirror. (**D**) A typical phase stability measurement of the whole system in 1 hour. (**E**) Diagram of the measured 5000 amplitudes of the matrix. (**F**) Diagram of the measured 5000 phases of the matrix.

Verification: 1

Classical spoofing strategies:

- Thermal states as input: excessive photon loss.
- Distinguishable SMSS: from mode mismatch.

Sampling with these inputs turns can be solved efficiently classically (since the density can be calculated efficiently.) Visible in the plots.

Verification: 2

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- If $r_{\mathrm{HOG}} \rightarrow 1$ as more samples are accumulated, the data look much more like ideal GBS than the mockup; thermal/mockup samples stay near 0.

Results

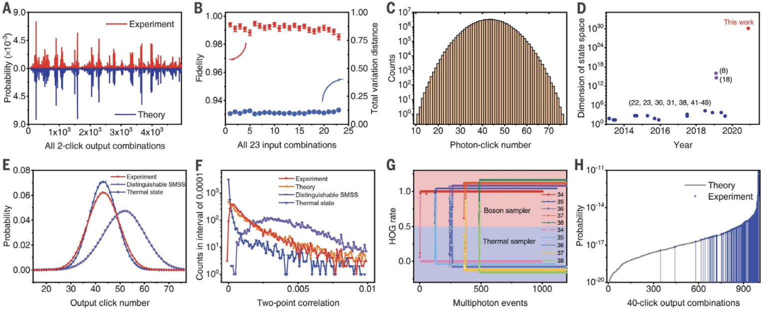

Fig. 3. Experimental validation of the GBS setup. (**A**) Experimental (red) and theoretical (blue) two-photon distribution with three TMSSs input. (**B**) Summary of statistical fidelity and total variation distance of two-photon distribution for 23 different input sets. (**C**) Output photon number distribution with all 25 TMSSs input. The average detected photon number is 43; the maximal detected photon number is 76. (**D**) Summary of the output state-space dimension. (**E**) Photon number distributions of the experimental result (red) and from the thermal state (blue) and distinguishable SMSS (purple) hypotheses. The deviations of the line shape and peak positions indicate that our experiment is far from these two hypotheses. (**F**) Two-photon correlation statistics for all two-mode combinations. The statistic of the experimental results (red) highly overlap with the theoretical predictions (orange) and deviate from the thermal state hypothesis (blue) and the distinguishable SMSS hypothesis (purple). (**G**) Validation against thermal state hypothesis with detected photon number ranging from 34 to 38. (**H**) Validation against uniform distribution.

Thank you!

Please pass me 🐵