On the practicality of quantum sieving algorithms for the shortest vector problem

João F. Doriguello, George Giapitzakis, Alessandro Luongo, Aditya Morolia[1]

May 2, 2026

[1]Center for Quantum Technologies, Singapore

How Peter Shor's Algorithm Dooms RSA Encryption to Failure

In 1994, Peter Shor created an algorithm for a theorical computer that solved a nearly impossible problem. Now that technology is catching up, Shor's algorithm guarantees the end to RSA Encryption.

Updated: Mar 12, 2025 06:40 AM EST

BLOG FEATURED CONTENT QUANTUM COMPUTING

Quantum computing and the future of cryptography: Understanding the imminent threat

Post-Quantum

Shor's Algorithm: A Quantum Threat to Modern Cryptography

Marin Ivezic · October 18, 2017 · 48 minutes read

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Google's new whitepaper says it could take only minutes for a quantum system to crack Bitcoin.

ANAND SINHA • 1 HOUR AGO

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Post-quantum lattice-based cryptography

History of Selected Algorithms Updates

Selected Algorithms: Key-Encapsulation Mechansims

Algorithm	Algorithm Information	Submitters	Comments
CRYSTALS-KYBER (2022) FIPS 203 **PQC License Summary & Excerpts**	Zip File (7MB) IP Statements Website	Peter Schwabe Roberto Avanzi Joppe Bos Leo Ducas Eike Kiltz Tancrede Lepoint Vadim Lyubashevsky John M. Schanck Gregor Seiler Damien Stehle Jintai Ding	Submit Comment View Comments

Selected Algorithms: Digital Signature Algorithms

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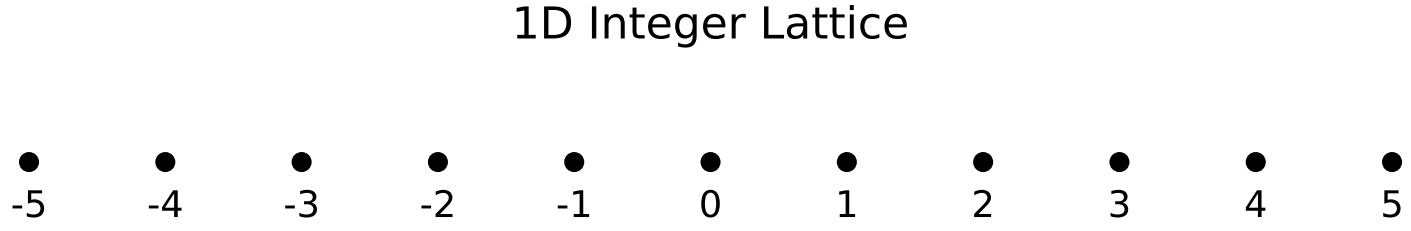

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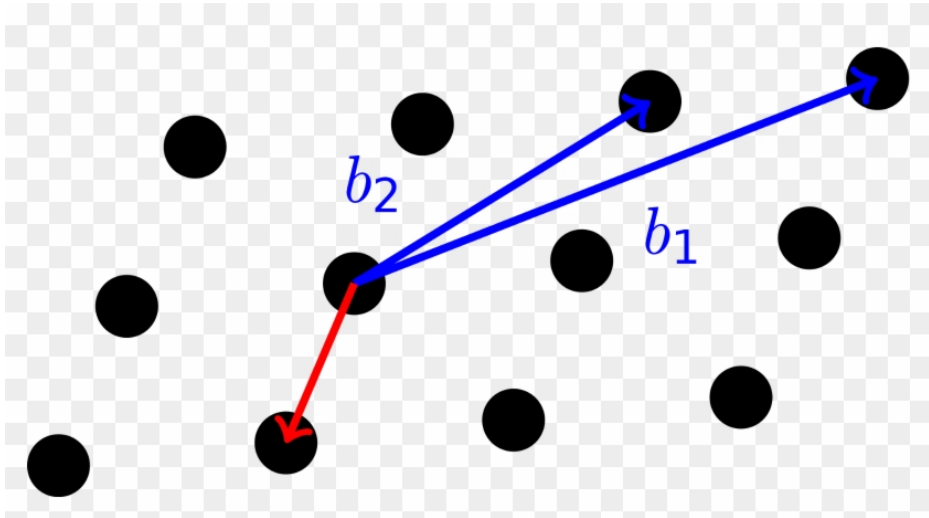

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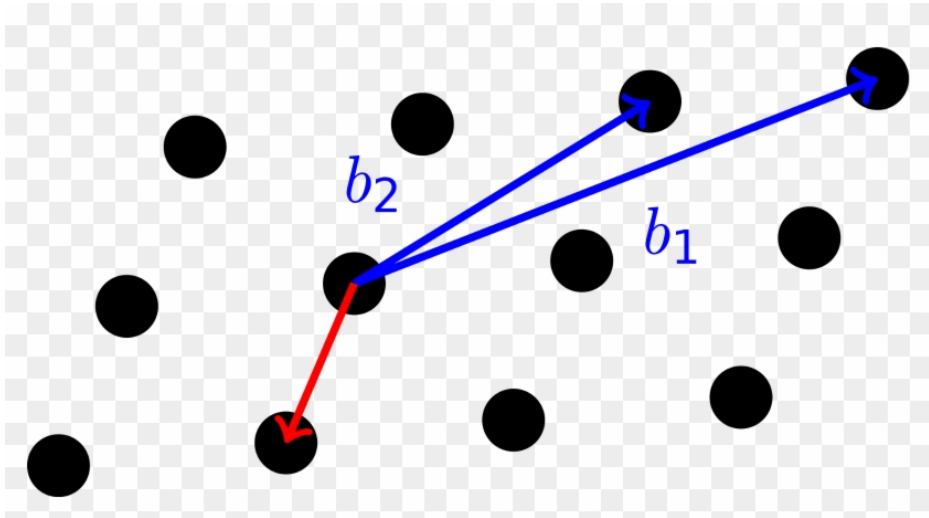

- Believed to be hard for quantum computers

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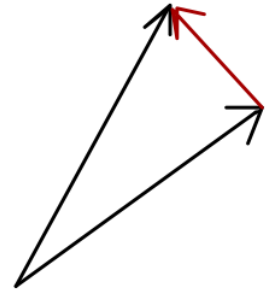
- Use approximate or exact SVP algorithm with the BKZ basis reduction algorithm.
- Asymptotic quantum speedups are known using Grover's search.
- What happens when you actually instantiate the Grover oracle, account for the QRAM and error correction?

The Algorithm: Lattice sieving. Fastest in practice. Poorly understood theoretically. Let's pretend we are cryptographers 😏

HALL OF FAME

Position	Dimension	Euclidean Norm	Seed	Contestant	Solution	Algorithm	Subm. Date	Approx. Factor
1	210	3808	0	Jintai Ding, Ziyu Zhao	vec	Sieving	2026-01-1	1.04497
2	200	3723	0	Ziyu Zhao, Jintai Ding	vec	Sieving	2025-03-4	1.04865
3	190	3613	0	Yao Sun and Shuai Chang	vec	Sieving	2024-07-21	1.04237
4	188	3582	0	Leizhang Wang, Lin Wang, Baocang Wang, Geng Wang, Huiwen Jia	vec	Sieving	2025-03-12	1.03823
5	187	3582	0	Shi Bai, Yuanmi Chen, Jiangxia Ge, Yituo He, Zhuo Huang, Juanru Li, Xiangxue Li, Chao Sun, Yu Yu	vec	Sieving	2026-01-20	1.04132

Lattice Sieving: [AKS01, NV08, MV10]



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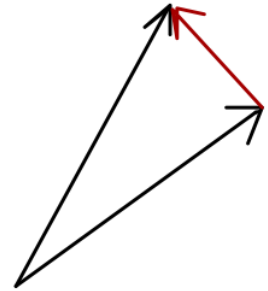

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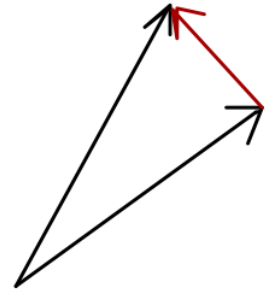

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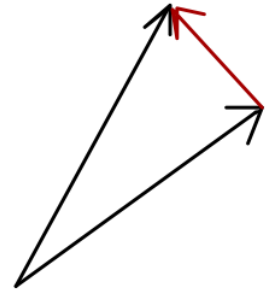

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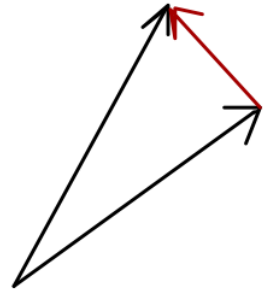

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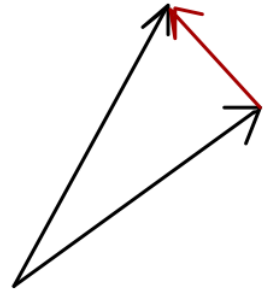

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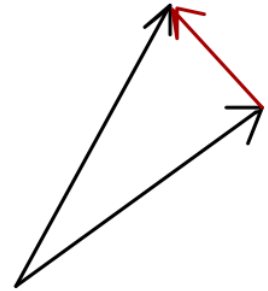

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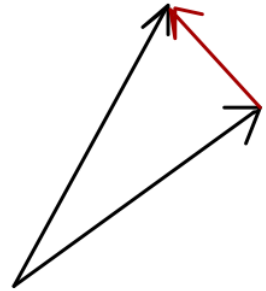

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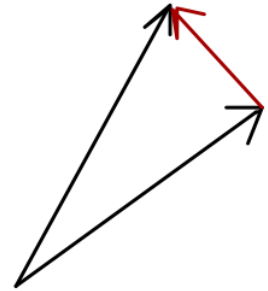

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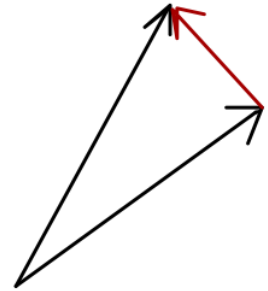

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Instantiate the oracle? Beyond asymptotics? What about dimension, say, 400? Probably more important for quantum computers?? Tech not so mature yet.

Quantum speedups for SVP? – Prior Work

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- Minimal speedup in cryptographically relevant dimensions.

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- Concrete physical architectures: nearest neighbor connectivity (Google's Sycamore) and active volume architecture [LN22] with concrete error correction model.
- Result: Little to no quantum speedup in dimension 400 for GaussSieve.

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- Reaction time: duration required to perform a layer of measurements, process the outcomes through a classical decoder (using algorithms such as minimum-weight perfect matching or union-find), and transmit new instructions back to the quantum hardware.
- Reaction depth: Number of such reactive measurements.
- Active volume: Counts the cost of expensive operations such as Toffoli gates and magic state distillation.

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- Two's-complement representation with $\kappa = 32$ (qu)bits.
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- Addition: Gidney's out-of-place quantum adder (Quantum'18).
- Comparison = addition.
- Multiplication: novel quantum circuit based on schoolbook multiplication with lower Toffoli-count (similar to recent work by Daniel Litinski arXiv:2410.00899).

Circuit / Resource	Toffoli-count	Toffoli-width	Reaction depth	Qubit-width	Active volume
Adder/Comparator	$\kappa - 1$	1	$2(\kappa - 1)$	3κ	$(\kappa - 1)(39 + C_{\lvert CCZ\rangle}) + 7$
Controlled adder	$2\kappa - 1$	κ	2κ	$4\kappa + 1$	$(\kappa - 1)(51 + C_{\lvert CCZ\rangle}) + 19$
Multiplier	$\kappa^2 - \kappa + 1$	$0.5\kappa^2 + 0.5\kappa$	$2\kappa \log_2 \kappa - 2\kappa - 2\log_2 \kappa + 4$	$2\kappa^2 + \kappa$	$28\kappa^2 - 42\kappa + 28$ $+(\kappa^2 - \kappa + 1)\, C_{\lvert CCZ\rangle}$
Multiplier (hybrid)	$0.5\kappa^2 - 1.5\kappa + 1$	0.5κ	$2\kappa \log_2 \kappa - 2\kappa - 2\log_2 \kappa + 2$	$1.5\kappa^2 + 0.5\kappa$	$20.25\kappa^2 - 48.75\kappa + 32$ $+(0.5\kappa^2 - 1.5\kappa + 1)\, C_{\lvert CCZ\rangle}$

Grover's Search

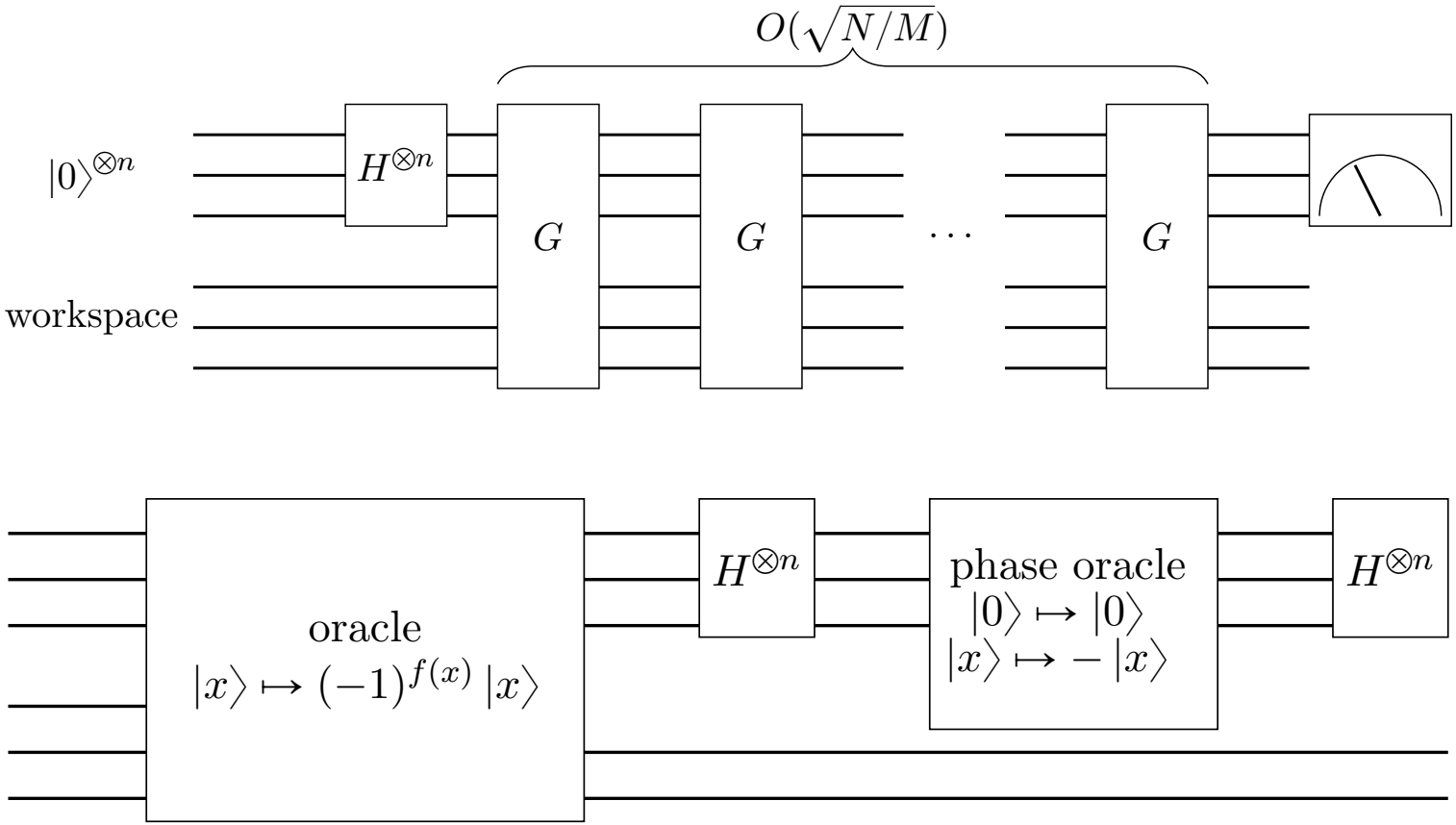

Figure: Circuit for Grover's search algorithm (top) and the Grover oracle G (bottom).

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- Exponential Grover's search is required [BBHT98].
- Precise resource estimate of exponential Grover's search was done by Cade, Folkertsma, Niesen, and Weggemans (Quantum'23).

Inside the Oracle

For GaussSieve we need two Grover oracles $|x\rangle \mapsto (-1)^{f(x)} |x\rangle$:

1. Find $i \in L$ such that $\|\vec{v} \pm \vec{w}_i\| < \|\vec{v}\| \iff \vec{w}_i \cdot (\vec{w}_i \pm 2\vec{v}) < 0$.
2. Find $i \in L$ such that $\|\vec{v} \pm \vec{w}_i\| < \|\vec{w}_i\| \iff |\vec{v} \cdot \vec{w}_i| \geq \|\vec{v}\|^2 / 2$.

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Fix $\vec{v} \in L$. The Grover oracle $|i\rangle \mapsto (-1)^{f_{\text{gauss}}(i)} |i\rangle$ is constructed as:

$$\begin{aligned}
|i\rangle |0\rangle^{\otimes \kappa(1+3D)} &\xrightarrow{\text{QRAM}} |i\rangle |\vec{w}_i\rangle |0\rangle^{\otimes \kappa(1+2D)} \\
&\xrightarrow{D \text{ adders}} |i\rangle |\vec{w}_i\rangle |\vec{w}_i - 2\vec{v}\rangle |0\rangle^{\otimes \kappa(1+D)} \\
&\xrightarrow{D \text{ multipliers}} |i\rangle |\vec{w}_i\rangle |\vec{w}_i - 2\vec{v}\rangle \left(\bigotimes_{j=1}^{D} |(\vec{w}_i)_j (\vec{w}_i - 2\vec{v})_j\rangle \right) |0\rangle^{\otimes \kappa} \\
&\xrightarrow{D-1 \text{ adders}} |i\rangle |\vec{w}_i\rangle |\vec{w}_i - 2\vec{v}\rangle \left(\bigotimes_{j=1}^{D} |(\vec{w}_i)_j (\vec{w}_i - 2\vec{v})_j\rangle \right) |\vec{w}_i^{\top} (\vec{w}_i - 2\vec{v})\rangle .
\end{aligned}$$

Sieve/Operations	QRAM	Adders	Multipliers	Extra CNOTs
`NVSieve`	1	$2D$	D	0
`NVSieve` + LSH/LSF	1	$2D$	D	0
`GaussSieve`	1 1	$4D-2$ $D+1$	$2D$ D^*	$2D\kappa+4$ 4
`GaussSieve` + LSH/LSF	1 1	$4D-2$ $D+1$	$2D$ D^*	$2D\kappa+4$ 4

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- Allows oracle queries

$$\text{QRAM} : |i\rangle |0\rangle^{\otimes \kappa} |\overline{0}\rangle \mapsto |i\rangle |x_i\rangle |\text{garbage}_i\rangle , \quad \forall i \in [2^n].$$

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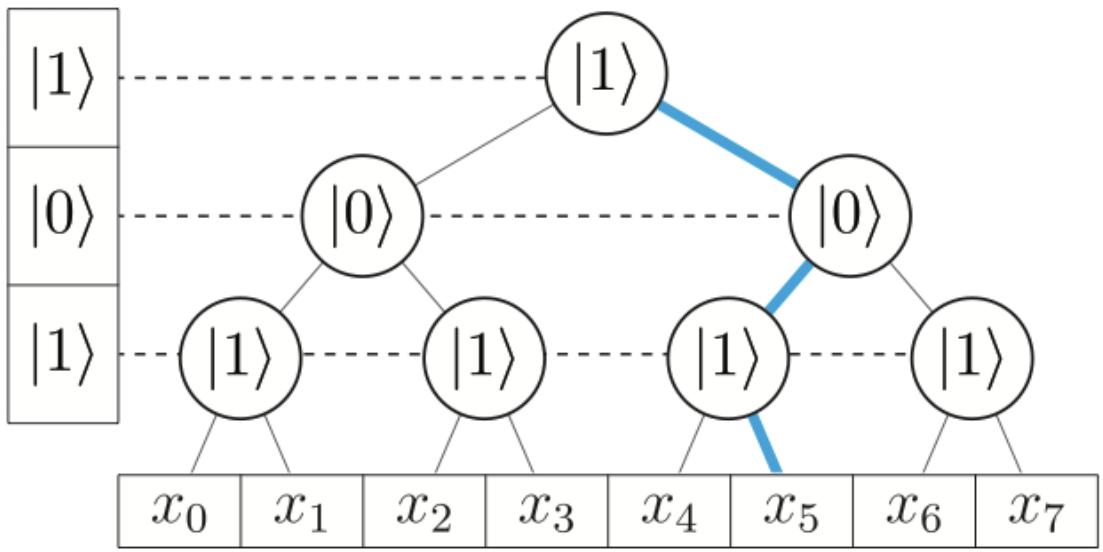
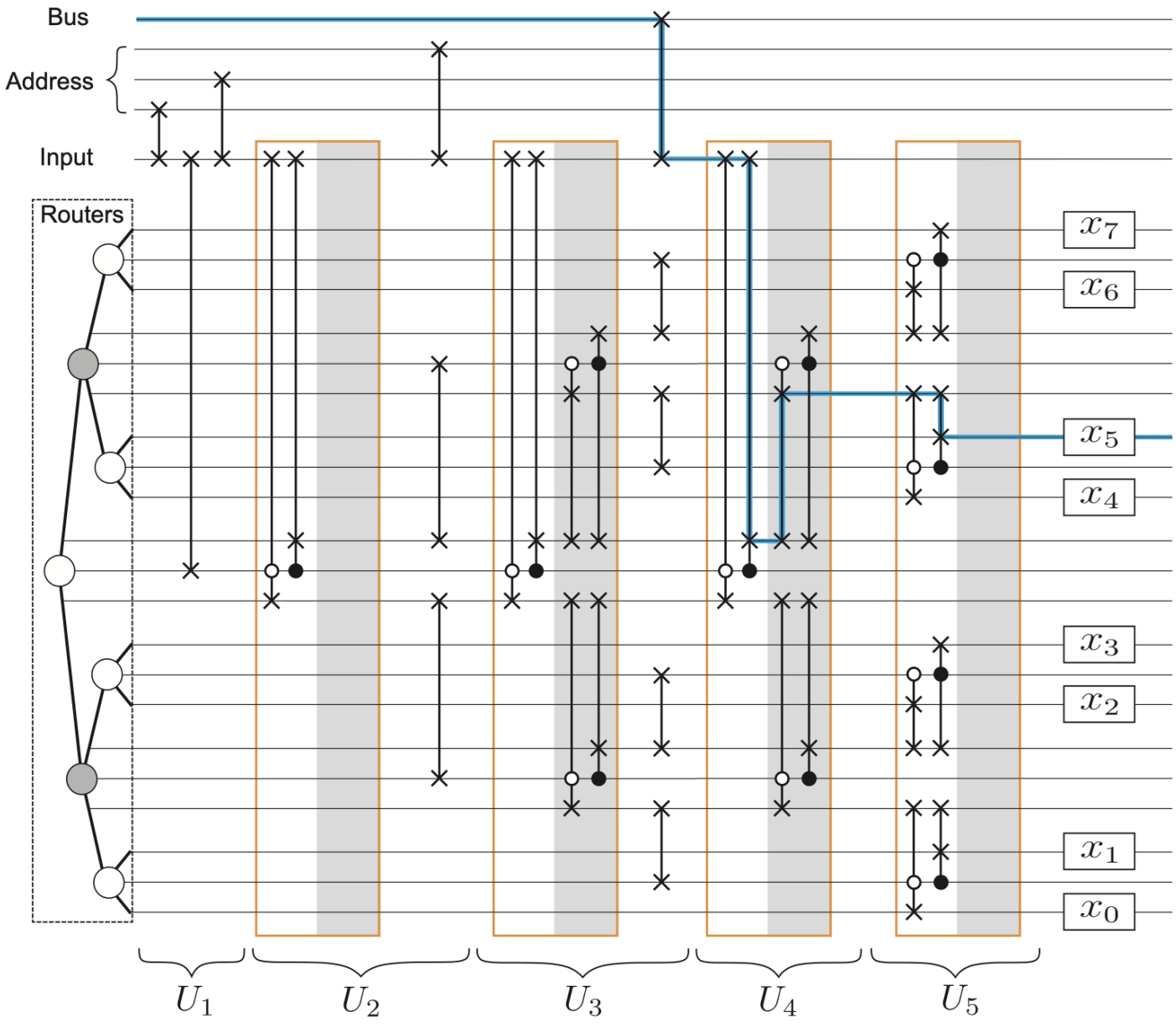
- QRAM is an external device that accesses a classical memory tape: Content is stored statically and can be classically rewritten without affecting the QRAM device.

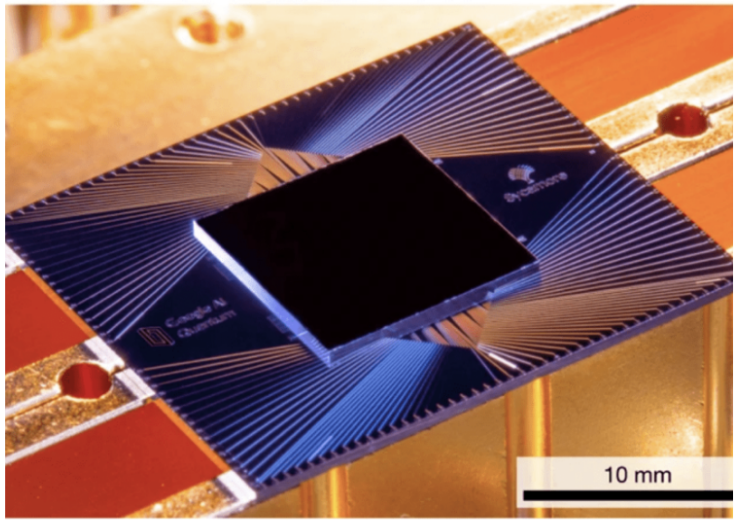


Pictures from Hann et al., PRX Quantum (2021)

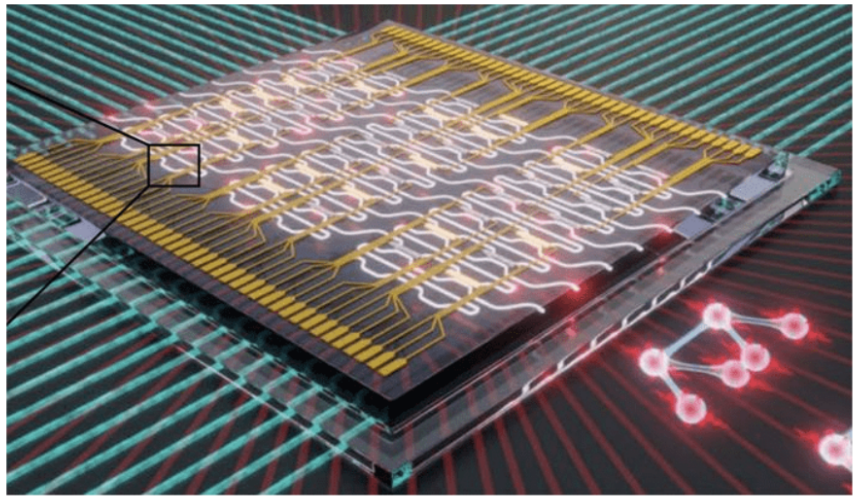
Quantum RAM 3



Quantum Error Correction 1



Baseline architecture



Active-volume architecture

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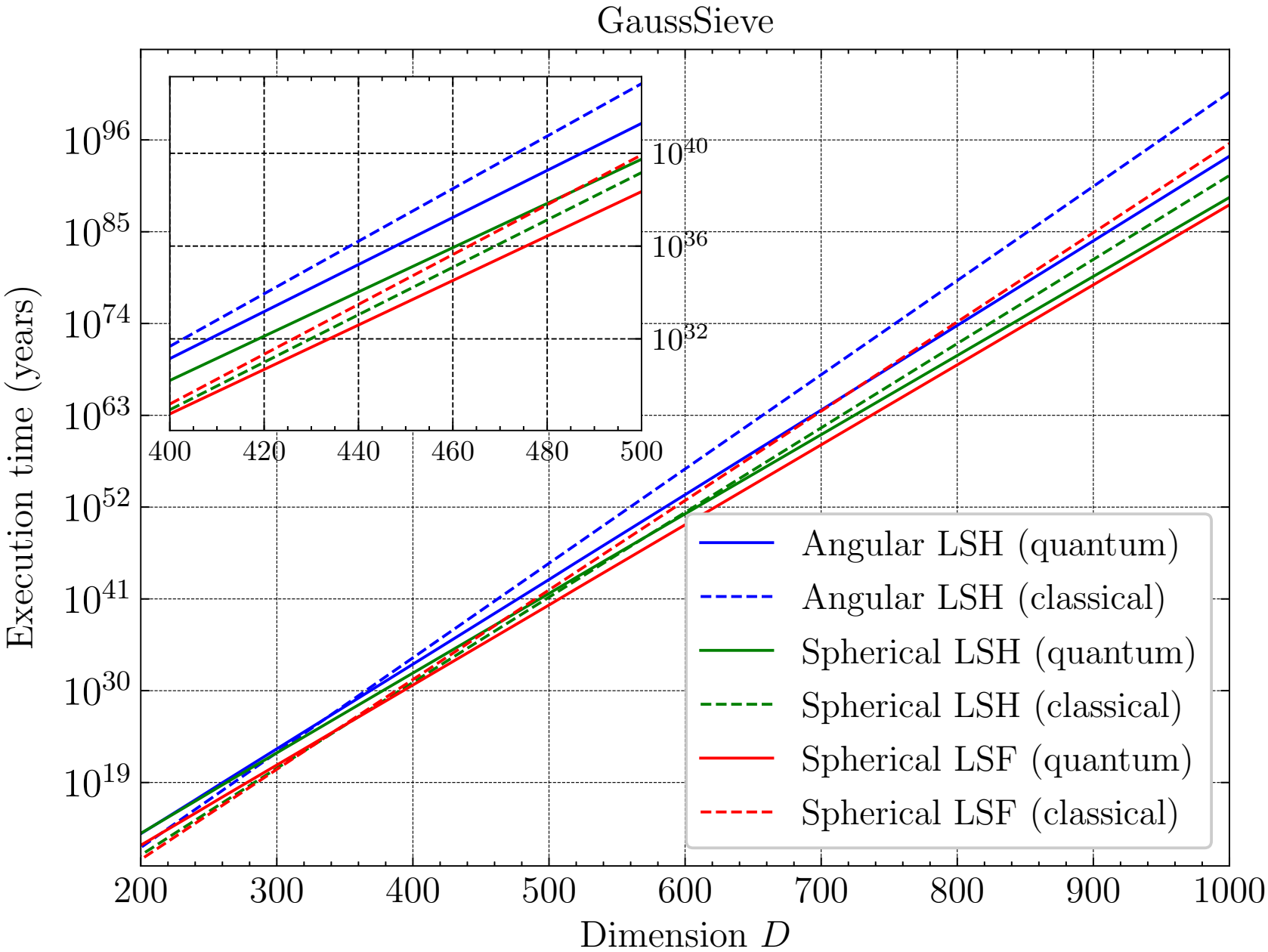
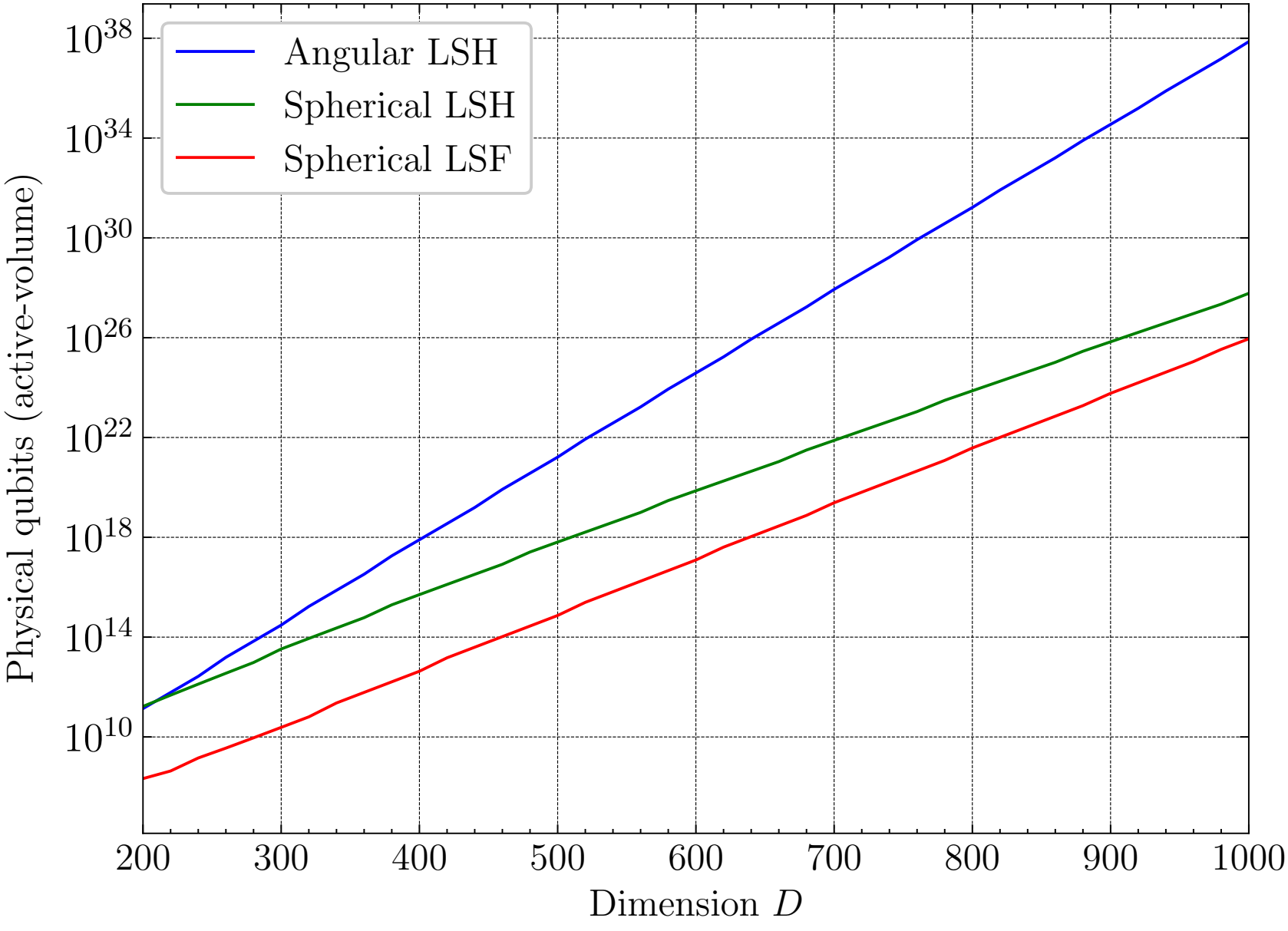
  - Reaction time of 1 μs (time to perform a layer of measurements, decode the outcome, and use the information to send new instructions).

Results 1


GaussSieve
Execution time (years)
10^{96}
10^{85}
10^{74}
10^{63}
10^{52}
10^{41}
10^{30}
10^{19}
10^{40}
10^{36}
10^{32}
400
420
440
460
480
500
200
300
400
500
600
700
800
900
1000
Dimension D
Angular LSH (quantum)
Angular LSH (classical)
Spherical LSH (quantum)
Spherical LSH (classical)
Spherical LSF (quantum)
Spherical LSF (classical)

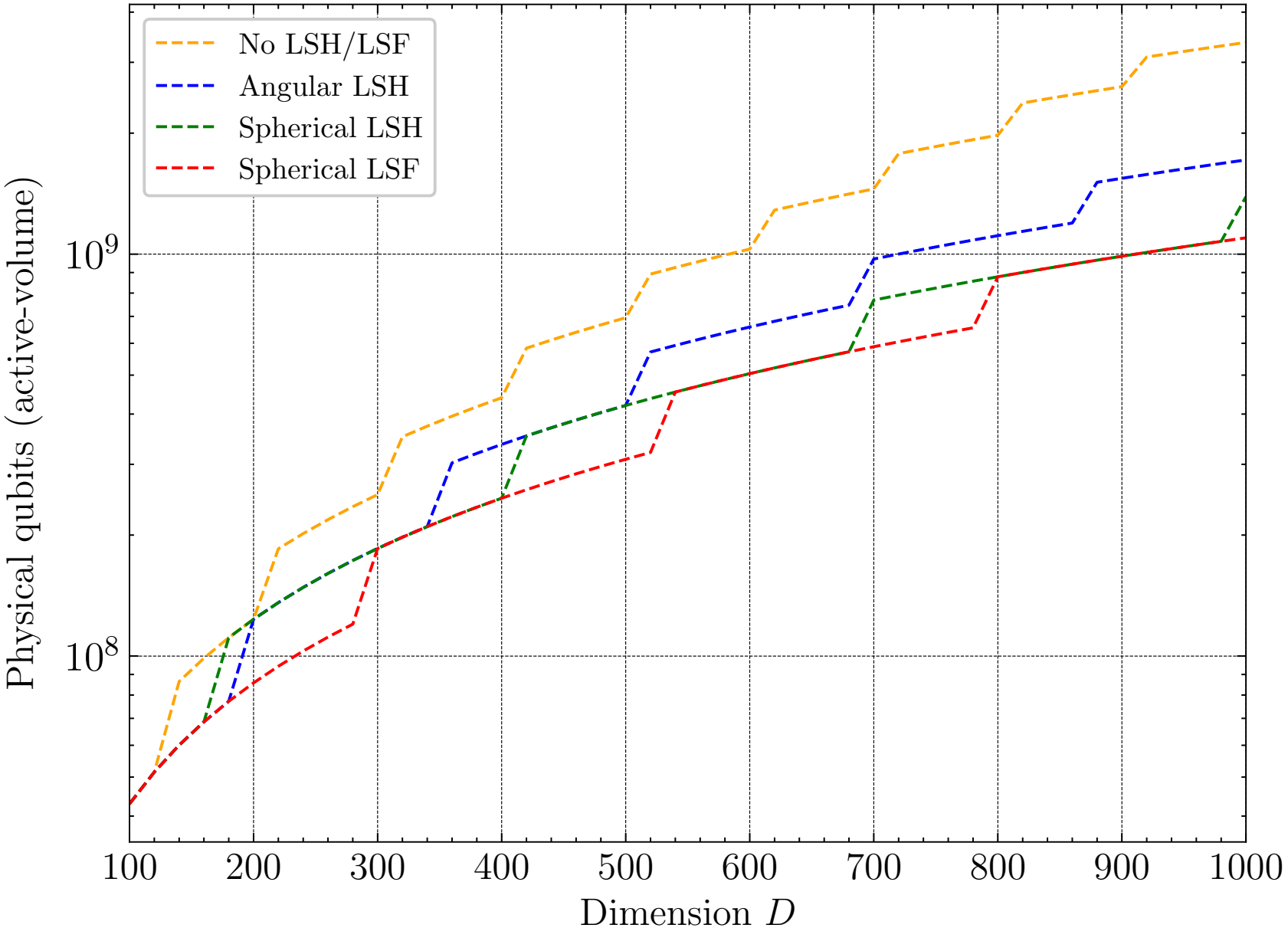
Results 2


Angular LSH
Spherical LSH
Spherical LSF
Physical qubits (active-volume)
10^{38}
10^{34}
10^{30}
10^{26}
10^{22}
10^{18}
10^{14}
10^{10}
200
300
400
500
600
700
800
900
1000
Dimension D

Results: Takeaways

- At dimension $D = 400$ (first NIST Security level), the best quantum sieve requires $\approx 10^{13}$ physical qubits to solve SVP in $\approx 10^{31}$ years.
- A 6-GHz-clock-rate single-core classical computer requires roughly the same amount of time.
- At dimension $D = 400$, the circuit time comes mostly from the quantum arithmetic circuits.
- At dimension $D = 1000$ (third NIST Security level), Grover's search provides a speedup by roughly five orders of magnitude.
- 99.9999% of physical qubits are used by the QRAM.

Results: 4


No LSH/LSF
Angular LSH
Spherical LSH
Spherical LSF
Physical qubits (active-volume)
10^9
10^8
100
200
300
400
500
600
700
800
900
1000
Dimension D

Future Directions

- Classically easy to use the notion of 'bits of security' to measure security. Quantum speedups are deceptive, composing them is tricky! Especially with QRAM. Can use tools like Google's Qualtran.

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- Classically easy to use the notion of 'bits of security' to measure security. Quantum speedups are deceptive, composing them is tricky! Especially with QRAM. Can use tools like Google's Qualtran.
- (Classical) algorithms poorly understood. Where are the mathematical guarantees? 🙄

Future Directions: 2

- Currently, there is little to no quantum speedup in the dimensions of cryptographic interest.
- Several possible improvements:
  - New magic state distillation protocols (Gidney, Shutty, Jones, Magic State Cultivation, QIP'25);
  - New QRAM circuits (Mukhopadhyay, Scientific Reports'25);
  - Better integration between QRAM and quantum error correction (Dalzell et al., arXiv'25);
  - QRAM calls parallelisation;
  - Consider quantum-random-walk-based sieving algorithms (requires non-asymptotic quantum random walk analysis akin to Cade et al.).

Thank you! 😀

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